TMX Montréal Exchange

CGZ™
Two-Year Government of Canada Bond Futures

CGF™
Five-Year Government of Canada Bond Futures

CGB™
Ten-Year Government of Canada Bond Futures

LGB™
30-Year Government of Canada Bond Futures

OGB™
Options on Ten-Year Government of Canada Bond Futures

Reference Manual

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1. Introduction

In September 1989, the Montréal Exchange (MX) launched the Ten-Year Government of Canada Bond Futures (CGB). Since the introduction of its CGB, MX continued developing the Canadian yield curve by launching the Two-Year (CGZ), Five-Year (CGF) and 30-Year (LGB) Government of Canada Bond Futures. Accompanying the 10-year futures contracts are the options on CGBs (OGB), adding more flexibility to managing interest rate risk. Through different applications of bond contracts, investors are able to both extract and preserve the value of capital pledged without term credit concerns.

The ten-year derivative product became popular with many bond dealers as it offers opportunities for portfolio enhancement and for use as a risk management tool. Asset managers have expanded their investment strategies to include the use of futures to extend duration, hedge anticipated interest rate moves and have gone so far as to create investment funds dedicated solely to the total return of CGBs.

The objective of this document is to explain how Government of Canada bond futures are used. Its purpose is to familiarize the public with the flexibility offered by the various interest rate products representing Government of Canada bonds. MX offers pure price competition, via SOLA, allowing each participant equal access to the best available price.

2. The Government of Canada Bond Market

The Government of Canada first issued bonds to the public in the 1940s to subsidize the war effort. These "Victory Bonds" have since matured, and their principal repaid. However, financing the ongoing fiscal requirements of the Government of Canada has required continuous issuances of bonds. The Bank of Canada on behalf of the Government issues Government of Canada marketable securities (bonds) by auctioning their debt on prearranged auction dates to the highest bidders.

Bond auction policies change from government to government, including new types of issues and eliminating others. Currently, the Federal Department of Finance has established five different maturities of bond issuances and one issue of real return bonds per calendar quarter. The 2-, 3-, 5-, 10- and 30-year auction dates, the amounts, settlement dates and other details are announced quarterly. The Bank of Canada auctions these issues to primary distributors of Government of Canada marketable bonds.

3. Government of Canada Bond Futures Contracts (CGZ, CGF, CGB, LGB)

What is a bond futures contract?

A bond futures contract is an agreement traded on an exchange that obligates the contracting parties to buy or sell a fixed amount of bonds at a future date, but at a price agreed upon in advance. It is entered into by two different parties: the seller (the short) and the buyer (the long). Once a position has been taken in a futures contract, two alternatives are available. On one hand, the contract will be held until expiry, when the short will have to make delivery to the long, who will take delivery of an eligible bond[1] at a price established in advance. On the other hand, the contract may be closed by taking the opposite position in the same contract.

Who uses bond futures contracts?

Bond futures contracts are used for hedging (risk management), speculating (trading for income generation) and arbitrage (profit from market anomalies).

Hedging consists of operations that minimize or eliminate risk arising from the fluctuations of an underlying bond or any security having similarities with these bonds (i.e. yield, maturity). A buy position or a "long" position in an underlying bond or security can be covered by a sell position or "short" position in the futures. Conversely, a "short" position in the underlying bond can be covered by a "long" position in futures. The greater the correlation between the two, the better the hedge will be. Therefore, the loss in one market will be partially or possibly entirely offset by the gains in the other.

Speculators aim to take profit from potential moves in the market. They look for trends in the market and position themselves accordingly. Most of the time they try to maximize their profits in the shortest period of time (intraday) but some hold their position for longer periods defined by the trend. These speculators benefit from an appealing financial leverage where great profits can be obtained if they make a correct prediction of the trend but on the other hand, the losses from an incorrect prediction can be just as big. Therefore, they must exercise great discipline in their speculative trading.

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[1] MX publishes the list of eligible or deliverable bonds known as the basket.

Arbitrage operations are aimed at profiting from pricing anomalies in the market (i.e. underlying bond vs. the futures or options vs. the futures). Price anomalies usually exist for a very short time; they are a result of the inefficiency of prices and are quickly corrected by the arbitrageur. To be effective, these trades must lock in an immediate profit, have no risk of incurring a loss and necessitate no net investment.

Bond futures are a very powerful tool to anyone wanting to:

- manage risk associated with Canadian content in a portfolio;
- enhance profit with Canadian content;
- add Canadian content to diversify a portfolio;
- speculate on the direction of the Canadian market;
- increase or decrease the duration of a fixed income portfolio;
- immunize against Canadian interest rate volatility.

Therefore, they are used by pension funds, domestic and foreign brokers (bond dealers), domestic and foreign banks, portfolio managers, hedge funds, insurance companies, finance and leasing companies, investment funds and individual investors. Also, non-financial companies benefit from trading bond futures because of their everyday involvement in financial markets (i.e. governments, electricity companies, car manufacturers, public companies).

4. Pricing of Bond Futures

Conversion factor

As with other bond futures contracts, the CGB allows the seller to fulfill delivery obligations with one of the different bond issues which fit the delivery standards of each contract. The price of each deliverable bond will be calculated through the use of a conversion factor.

The conversion factor allows for the comparison of the deliverable Government of Canada bonds (with their varying coupons and maturities) on a common basis. It is calculated by determining the price at which a deliverable bond would have a semi-annual yield equal to the notional coupon.

The formula is as follows:

$$F = 1/(1.03)^d \times [c/2 + c/0.06 \times (1 - 1/1.03^n) + 100/1.03^n] - c/2 \times (1 - d)$$

Where:
F = conversion factor;
c = coupon on C$100 face amount;
n = number of half years from the first day of the futures delivery month to the final maturity date of the bond;
d = the fractional part of n determined (after rounding down to the nearest whole three-month period[2]) as the number of whole three-month periods divided by six months, i.e. 0.0 or 0.5.

The notional coupon of the CGB contract is expressed as the parameters 0.06 and 1.03 (for compounding semi-annually) which flow throughout the formula.

[2] The conversion factor for a deliverable bond is calculated in complete half-year periods from the first day of the futures contract month to the maturity date of the bond, with the number of excess months calculated in complete three-month periods (in the case of the CGB and LGB futures contracts) or in complete one-month periods (in the case of the CGZ and CGF futures contracts). Refer to contract specifications on pages 23-24 of the manual.

Here is an example of deliverable Canadian government bonds and their respectable conversion factors for different months:

GOVERNMENT OF CANADA BONDS		OUTSTANDING	CGB EXPIRATION MONTHS			
Coupon	Maturity	(CAN $ million)	June 2011	September 2011	December 2011	March 2012
3 ¾%	June 1, 2019	17,650	0.8587	s.o.	s.o.	s.o.
3 ½%	June 1, 2020	13,100	0.8281	0.8317	0.8354	0.8391
3 ¼%	June 1, 2021	9,000	0.7954	0.7992	0.8030	0.8069
TOTAL OUTSTANDING DELIVERABLE BONDS (C$ million)			39,750	22,100	22,100	22,100

Conversion factors computed with a yield equal to 6%

The lifeline of a contract

A futures contract is created when the first transaction is completed. The trade is then warehoused at a clearing firm who makes or receives variance payments to and from the clearing corporation[3]. Each market participant is liable to their clearing firm for the price action of their trades. The clearing firm is in turn liable to the clearing corporation.

Bond futures will trade in unison with the underlying cash bond market, factoring into the price the repo rate and time to expiry.

Delivery

There are five important dates[4] in a bond futures contract life:

- The first notice day is three business days (for CGF, CGB and LGB) prior to the first business day of the delivery month and is the first day a short futures position (seller) may announce his intention to deliver the underlying instrument (cash bond) to the holder of the long futures position. In the case of the CGZ, it is two business days prior.

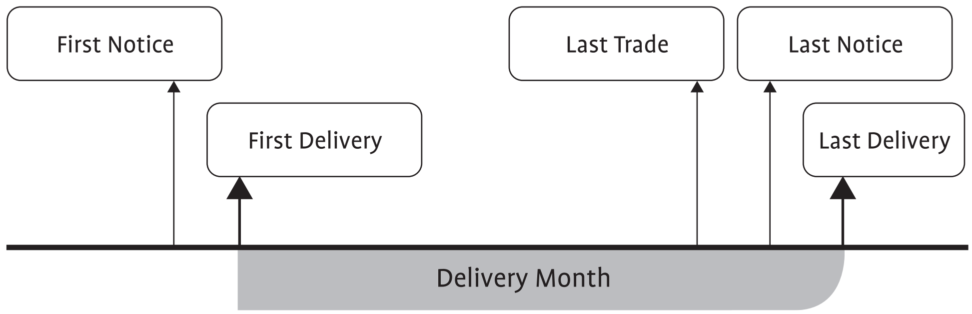
- A long futures position may receive delivery of the underlying cash bond at any time in the period between first delivery day (first business day of the delivery month) and last delivery day (last business day in the delivery month).

- The last trading day is seven business days prior to the last business day of the delivery month and is the final trading day of the futures contract. Thereafter, the delivery of all cash bonds will be set against the settlement price of the futures which is set at 1:00 p.m. on the last trading day.

[3] The main clearing corporations are financial establishments with very good capitalization. By implication, their risk of insolvency is very low. In Canada, the Canadian Derivatives Clearing Corporation (CDCC) acts as the central clearing counterparty for financial instruments traded on MX.

[4] MX publishes expiration calendars for all derivative financial instruments offered. You can download a copy from the website at www.m-x.ca

- The last notice day is three business days (for CGF, CGB and LGB) prior to the last business day of delivery month and is the last day a short futures position (seller) may announce his intention to deliver the underlying instrument (cash bond) to the holder of a long futures position. In the case of the CGZ, it is two business days prior.



Cheapest-to-deliver (CTD)

The cost of delivery of the bonds is influenced by both shifts in the yield curve and the fact that these same bonds trade more or less expensive than the yield curve would indicate. Therefore, the delivery price supplied by the conversion factor for each of the bonds in the delivery basket will tend to differ from its market price. This difference results in one of the bonds in the basket having less losses or greater gains than any of the other deliverable bonds. This bond is known as the cheapest-to-deliver (CTD).

The short will naturally choose to deliver the bond that is the least expensive or the cheapest-to-deliver. As a result, the futures price will tend to track the price of the CTD more closely than that of other bonds in the delivery basket. Determining the CTD calls for an understanding of the relationship between cash and futures markets.

1. Cash and futures prices

The conversion factor for a specific bond, during a specified period, remains constant in spite of changes in the price of the bond or the futures and represents a common point linking cash and futures markets. Thus, a simple operation with the conversion factor allows the futures price to be expressed in terms of cash prices or vice versa, in the following formulas:

Futures price expressed as a cash price:

$$\textbf{Cash equivalent price} = \textbf{Futures price} \times \textbf{Conversion factor}$$

Cash price expressed as a futures price:

$$\textbf{Futures equivalent price} = \frac{\textbf{Cash price}}{\textbf{Conversion factor}}$$

To simplify comparisons between the futures price and the different deliverable bond issues, we will use the futures equivalent price in the following discussions. The results will therefore not be in dollar values but can be transformed easily by multiplying the results by the conversion factor of the bond for which the calculation was done.

2. Theoretical futures price: Cost of carry

For bond futures, in order to guarantee the delivery of a cash bond without any risk, the seller must purchase the bond at the moment the futures contract is established, and hold it until delivery. This involves the short-term cost of holding and financing the bonds until delivery, and the long-term yield received from the bonds during the same period. Therefore, prior to delivery, the futures equivalent price will have to be adjusted for the difference between the interest accrued from the coupon payments and the short-term financing rate (repo rate) for the period the position is held. The difference in the long-term yield and the short-term cost is known as cost of carry and is expressed as follows:

$$\text{Cost of carry} = \text{Coupon income} - \text{Financing cost}$$

More explicitly:

$$\text{Cost of carry} = (AD - AS) - (MV \times r \times t)$$

Where:
- **AS** = accrued interest on the deliverable bond on the date the position is initially established;
- **AD** = accrued interest on the deliverable at delivery, including coupons received since settlement and reinvested at the financing rate;
- **MV** = market value (price + AS) of the deliverable bond;
- **r** = financing rate (T-bill rate);
- **t** = term (in years) from the date when the position is initially established to the delivery date.

In order to express the dollar value cost of carry of the CTD in terms of the futures price, it must be divided by the conversion factor of the CTD. We then obtain the adjusted cost of carry:

$$\text{Adjusted cost of carry}_{CTD} = \frac{\text{Cost of carry}_{CTD}}{f_{CTD}}$$

Where : **f** = conversion factor of the bond

Given the cost of carry, the theoretical equilibrium price of the bond futures relative to its underlying cash bond market price is the following:

$$\text{Theoretical futures price} = \text{Futures equivalent price}_{CTD} - \text{Cost of carry}_{CTD}\text{ (adjusted)}$$

Or, alternately:

$$\text{Cost of carry}_{CTD}\text{ (adjusted)} = \text{Futures equivalent price}_{CTD} - \text{Theoretical futures price}_{CTD}$$

3. The basis

Although the price of a futures contract will closely track the futures equivalent price of the CTD, a difference will be observed between the two. This difference is known as the basis and can be expressed as follows:

$$\text{Basis}_{CTD} = \text{Futures equivalent price}_{CTD} - \text{Actual futures price}$$

The basis may also be expressed in terms of the cash bond, i.e. in dollar value:

$$\text{Basis}_{CTD} = \text{Cash bond price}_{CTD} - (\text{Actual futures price} \times f_{CTD})$$

In theory, the basis should be equal to the cost of carry, as stated by the theoretical futures price described previously. Thus, in theory, the investor should be indifferent to holding the long cash or short futures position, as the cost and yield would be the same.

The cost of carry will depend on the orientation of the yield curve. In the event of an upwardly sloping (normal) yield curve, the holder of the bonds receives more income than the cost of financing, resulting in positive carry. In the case of a downward sloping (inverted) yield curve, the holder of the bond receives less income from holding the bonds than the cost of financing, which results in negative carry.

Given the relationship linking the cost of carry to the futures prices, a normal yield curve will result in the futures price being below the cash bond price. Alternately, under an inverted yield curve, the futures price will exceed the cash bond price in order to make up for the net loss on the bonds.

The price of futures will converge towards the price of the CTD as we near the contract expiry. This results from the cost of carry shrinking as the period during which the bond must be held diminishes. At expiry the cost of carry is zero.

Important concepts about the basis:

- "Long the basis" is: Long the Government of Canada bond, short the futures contract.

  The position represents owning an asset from the bond settlement day to the futures delivery day. In the cash market, it is known as a reverse repurchase agreement (reverse repo), which receives interest at the implied repurchase (repo) rate. A market participant agrees to purchase the CTD bond now and resell it in the future. This is an exchange of cash for bonds.

- "Short the basis" is: Short the Government of Canada bond, long the futures contract.

  The position represents owning a liability from the bond settlement day to the futures delivery day. In the cash market, it is known as a repurchase agreement (repo), which pays interest at the implied repurchase (repo) rate. A market participant agrees to sell the CTD bond now and repurchase it in the future. This is an exchange of bonds for cash.

4. Delivery options

The theoretical price we previously described is actually a simplification of the pricing of contract. In fact, bond futures prices will generally be slightly below the results expected by the cost of carry of the CTD. This is because the short in the futures also holds an implied put option.

The seller of the futures holds three options:

a. option to deliver or not;
b. option on which bond to deliver;
c. option to provide delivery notice between the closing of the markets at 3:00 p.m. and 5:30 p.m., during which time the underlying bond price may change with no effect on the futures settlement price. This option is referred to as a "wildcard".

5. Identifying the CTD

The bond that has greatest implied repo rate is cheapest-to-deliver.

Implied repo rate:

It is basically the effective rate earned by purchasing a cash bond and selling the futures, with the intention of delivering that particular bond in the future.

By using the actual futures price, the cash price of the bond, the coupon income and taking into account the accrued interest, we can determine that rate with the following formula :

$$IRR = \frac{(F \times f) + AD - MV}{MV \times t}$$

Where:
- **F** = futures settlement price;
- **f** = conversion factor;
- **AD** = accrued interest on deliverable at delivery, including any coupons received since settlement and reinvested at the financing rate;
- **MV** = market value of the deliverable bond (price + accrued interest);
- **t** = term (in years) from the date on which the position is initially established to the delivery date.

Another way of identifying the CTD is this rule of thumb.

RULE OF THUMB	
Government Yield to Maturity is:	**Cheapest to deliver will be the bond in the basket with:**
LOWER than the notional coupon of the futures contract. Futures is at a premium (> 100).	the **highest coupon** and **shortest maturity** (shortest duration).
HIGHER than the notional coupon of the futures contract. Futures is at a discount (< 100).	the **lowest coupon** and **longest maturity** (longest duration).

It is important to remember that this guideline is true most of the time but numerous scenarios exist where other bonds may be the cheapest to deliver. It is especially true when the underlying bond yield is near the notional coupon of the futures contract. Duration will be explained further on when we look at the use of bond futures.

6. Basis risk

The risk that the futures contract does not perfectly track the bond that is being hedged is known as basis risk. Basis risk is usually greater for bonds other than the CTD. As we have seen, futures contracts will generally converge towards the price of the CTD, especially in the final weeks prior to expiration.

Actual data on the basis tends to indicate that the futures contract will trade sometimes cheaper, and sometimes richer during the life of the contract. Consequently, even when adjusted for the delivery option, the basis will not always equal the adjusted cost of carry, i.e. the actual futures price is not always equal to the theoretical futures price. Therefore, hedgers using futures for terms which end on dates other than contract expiry will experience basis risk, even when hedging with the CTD. The loss due to the basis risk is, in fact, the same as the profit gained from arbitrage operations.

An astute hedger will aim to enter a position in order to profit from shifts in the basis. If possible, he will go short the futures when it is trading expensive (rich) and go long the futures when it is trading cheap.

Here are some general situations that may increase basis risk:

- changes in the slope of the yield curve;
- changes in yield spreads;
- changes in credit ratings;
- changes in the short-term rates used to evaluate the cost of carry.

Note that these need not actually occur. The futures price may react as strongly to market sentiments as it would to real changes for any one of these situations.

7. Change of the cheapest-to-deliver bond

In our study of the basis and comparison of the theoretical to the actual futures price, we considered that the cheapest-to-deliver bond remained unchanged. However, the CTD can change during the life of the contract. Users of bond futures must familiarize themselves with the effects of a change in the CTD on their investment positions.

Supposing an investor holds the CTD with the intention of delivering it into a short futures position. As seen, in this classic cash-and-carry situation the investor has locked in future profits. A change in CTD means that this is no longer the cheapest deliverable bond. The investor may benefit from this situation by selling his bonds, purchasing the new CTD and delivering it into the futures. In this way, the investor secures greater gains than originally expected.

Alternately, an investor may have originally hedged a bond that was not the CTD and later became the CTD. In this case, the investor will once again increase his returns simply due to the change in the cheapest-to-deliver bond.

Generally, the CTD may change as a result of various causes:

- changes in the overall level of interest rates in Canada;
- change in yield spreads;
- issue of a new eligible bond that becomes the cheapest to deliver.

A foreseen change in the CTD will impact on the delivery option. The higher the expectation of a change in the CTD, the higher the value of the delivery option, and the cheaper the actual futures price relative to its theoretical price.

5. Using Government of Canada Bond Futures

The hedge ratio

A hedge is generally defined as a transaction that reduces risk, usually at the expense of potential reward. Bond futures contracts are ideally suited to reduce interest rate risk over a specific period of time. A government bond futures hedge is achieved through the purchase or sale of an offsetting futures position in order to protect a position against interest rate risk.

The most important aspect of hedging with bond futures is the hedge ratio (the hedge ratio is directly influenced by the variation of the basis), which answers the question: how many futures contracts should be bought or sold? Because futures contracts and the position to be hedged often display different patterns of variation over time, the number of contracts necessary to offset the loss will tend to differ for each position. The hedge ratio is a measure of the relative price sensitivities of the futures contract (cheapest) and the position to be hedged, and is used to determine the necessary number of futures contracts to hedge a position.

Because arbitrage opportunities between the cash and the futures markets are generally achieved through cash-and-carry operations based on the CTD, the price of the futures will track, and converge towards, the CTD.

The hedge ratio must therefore reflect the price sensitivity of the CTD, as represented by the following formula:

$$\text{Hedge ratio} = \text{Relative price sensitivity} \times \text{Conversion factor}_{CTD}$$

and

$$\text{Number of contracts} = \text{Hedge ratio} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

There are four basic approaches used to determine the hedge ratio or the relative price sensitivities:

- conversion factors;
- relative price sensitivity or one basis point value (BPV);
- duration;
- regression analysis (yield beta).

Conversion factor hedge ratio

The price of a bond futures contract, adjusted by the conversion factor of the CTD, varies in direct relation to fluctuations in the price of the CTD. Therefore, the CTD can be hedged by applying the conversion factor as the hedge ratio. Because the relative price sensitivity in this case is one, the hedge ratio is the following:

$$\text{Hedge ratio} = \text{Conversion factor}_{CTD}$$

Our formula for the number of contracts thus becomes:

$$\text{Number of contracts} = \text{Conversion factor}_{CTD} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

Conversion factors may also be used for bonds other than the CTD. However, as the futures price tracks and converges towards the price of the CTD, the hedge will only be effective if both bonds react in the same way to interest rate changes.

Relative price sensitivity, or basis point value (BPV)

When the bond to be hedged differs from the CTD in terms of coupon or maturity, relative price sensitivity and basis point value provide an estimate to the hedge ratio. The relative price sensitivity will use the ratio of price changes of the bonds for an expected yield change. The basis point value method will use the ratio of price changes of the bonds for a one basis point shift (0.01%) in the yield of the bonds. For both these methods, attention must be given in the case of large changes in interest rates, which result not only in price changes, but also in changes in the interest rate sensitivity of the bonds. The calculation of the BPV will be detailed in the following point on bond duration.

In this case the hedge ratio is obtained as follows:

$$\text{Hedge ratio}_{BPV} = \left(\frac{\text{BPV}_{\text{Hedged bond}}}{\text{BPV}_{CTD}} \right) \times \text{Conversion factor}_{CTD}$$

Our formula for the number of contracts thus becomes :

$$\text{Number of contracts} = \text{Hedge ratio}_{BPV} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

Duration hedge ratio

1. Duration

Duration is a measure of the life of a bond (in years) as it relates to change in its price. Technically, it is the present value weighted time to maturity of the cash flows of a fixed payment instrument, such as a Government of Canada bond. The formula used to calculate the duration is the following:

$$\text{Macaulay duration} = \frac{\sum_{T=1}^{m} \frac{tC_t}{(1+r)^t}}{\sum_{T=1}^{m} \frac{C_t}{(1+r)^t}}$$

Where "t" is the period of each payment (that is 1, 2, 3,.....m), "$C_t/(1 + r)^t$" is the present value of each payment, and "m" is the number of periods. In order to calculate the present or discounted value of the payments in each period, it is common to use the yield to maturity of the bond.

The duration of a bond changes according to its coupon, its time to maturity and its price. The duration of a bond gives a measure of the sensitivity of the bond's price to a change in interest rates. The longer the duration of the bonds, the greater will be the change in prices as a result of interest rate changes, and the lesser the variance in the prices of short-duration bonds. A forecast of higher interest rates leads to a higher demand for short-duration bonds, whereas a forecast for lower interest rates (increase in prices) leads to the acquisition of long-duration bonds.

2. Modified duration

The equation for duration can be modified slightly in order to be used as a measure of volatility. This is known as modified duration and is calculated as follows:

$$\text{Modified duration} = \frac{\text{Macaulay duration}}{1 + y/p}$$

Where: y = yield to maturity of the bond (in decimal form);
p = number of periods per year;

or:

y/p = periodic yield (in decimal form).

The modified duration can be used to determine the optimal hedge ratio. The equation for the hedge ratio obtained using the modified duration is the following:

$$\text{Hedge ratio}_{\text{Modified duration}} = \frac{\text{Cash price of bond}}{\text{Cash price of CTD}} \times \frac{\text{Modified duration of bond}}{\text{Modified duration of CTD}} \times \text{Conversion factor}_{\text{CTD}}$$

The number of futures contracts necessary for the hedge is calculated as follows:

$$\text{Number of futures contracts} = \text{Hedge ratio}_{\text{Modified duration}} \times \left(\frac{\text{Face value of cash bonds}}{\text{Nominal value of bond futures}} \right)$$

In addition, coming back to our concept of BPV introduced in the previous point, once the modified duration is known, the price sensitivity to changes in interest rates is determined by the following:

$$\frac{\text{Change in price}}{\text{Price}} = -\text{Modified duration} \times \text{Change in interest rates}$$

The minus sign in the formula takes into account the fact that the price of bonds move inversely to interest rates. Dollar duration is a measure of the dollar price change resulting from a given change in interest rates, and is obtained by multiplying the result of the above equation by the price of the bond. Furthermore, by multiplying the above equation by the price of the bond and by a 0.01% change in interest rates, one obtains the basis point value (BPV) of the bond.

$$\text{BPV} = -\text{Modified duration} \times \text{Bond price} \times 0.01\% \text{ change in interest rates}$$

Regression analysis: Correlation coefficients and yield beta (β)

The methods used above to calculate the hedge ratio are based on the theoretical (duration and BPV) and conventional (conversion factors) considerations. Because theory and conventions fail to capture all the aspects of market price evolution, the hedge ratio may be adjusted by comparing actual historical market data on the price or yield evolution of the CTD and the instrument or position to be hedged. Through the application of statistical regression techniques, we may evaluate either the correlation coefficient or the yield, both of which may be used to obtain a more accurate hedge ratio.

The correlation coefficient may be used directly in the following manner:

$$\text{Hedge ratio} = \text{Correlation coefficient} \times \text{Conversion factor}_{\text{CTD}}$$

The yield beta is used to adjust the hedge ratio of a duration or basis point value (BPV) weighted hedge as follows:

$$\text{Hedge ratio} = \frac{\text{Cash price of bond}}{\text{Cash price of CTD}} \times \frac{\text{Modified duration of bond}}{\text{Modified duration of CTD}} \times \text{Yield beta} \times \text{Conversion factor}_{\text{CTD}}$$

Which is the same as:

$$\text{Hedge ratio}_{\text{BPV}} = \frac{\text{BPV}_{\text{Hedged bond}}}{\text{BPV}_{\text{CTD}}} \times \text{Yield beta} \times \text{Conversion factor}_{\text{CTD}}$$

6. Margins

Margin is made up of two parts: the initial margin and the maintenance margin. Upon entry into a futures position, the clearing corporation requires that market participants pledge a minimum amount of initial margin. This amount is held by an approved depository on the behalf of the market participant. As of May 2011, the initial margin was as follows:

MARGIN TYPE	CGZ	CGF	CGB	LGB
Speculator	C$1,400	C$1,450	C$2,100	C$3,200
Hedger	C$1,300	C$1,350	C$2,000	C$3,000
Spreads	C$200	C$200	C$200	C$200

As an example, a speculative buyer of 10 CGBM11 must post C$21,000 in initial margin.

10 contracts × C$2,100 per contract = C$21,000

Maintenance margin (or variation margin) is posted daily for differences between the market price and the transaction price. In example, if the market on the CGBM11 settled at $121.35, a buyer of 10 CGBM11 at a price of $121.40 would have to post $500 in maintenance margin.

Maintenance margin =

$$\frac{\text{Price change}}{\text{Minimum price fluctuation (0.01)}} \times \text{Number of contracts} \times \text{Value of the minimum price fluctuation (C\$10)}$$

$$\frac{(121.35 - 121.40)}{0.01} \times \text{10 contracts} \times \text{C\$10} = \text{-500 \$}$$

The following day, the position will be marked from the previous close to the next close. If the market of the CGBM11 the next day closed at $121.37 the original buyer would have to post another $200 dollars in maintenance margin.

$$\frac{(121.35 - 121.37)}{0.01} \times \text{10 contracts} \times \text{C\$10} = \text{-200 \$}$$

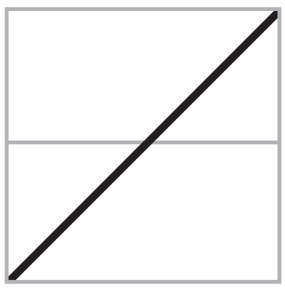
Conversely the seller of the CGBM11 may withdraw $300 of funds against their profitable position.

$$\frac{(121.40 - 121.37)}{0.01} \times \text{10 contracts} \times \text{C\$10} = \text{300 \$}$$

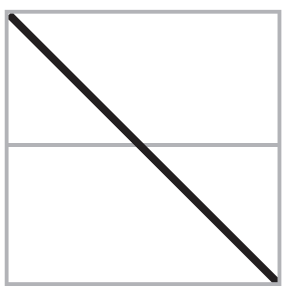
OGBs are options on the CGB which represent the right, but not the obligation, to buy (call) or sell (put) the CGB contract at a specified price (strike) in a specified amount of time (expiry). OGBs are an added risk management feature of the CGBs enabling market participants to modify a portfolio's market risk to increase profit potential or reduce losses.

Price action of the underlying

As noted previously, CGBs move as a function of the cheapest-to-deliver bond. The risk position of futures is straightforward in comparison to options. Futures have two intrinsic measures. The sensitivity of upward and downward movement in the CGBs is referred to as its delta. The delta of the CGBs is C$10 per tick (0.01) per contract. The second measure of sensitivity is a relative measure of the CGBs to the cheapest-to-deliver bond, which is the implied cost of carry or the futures basis.

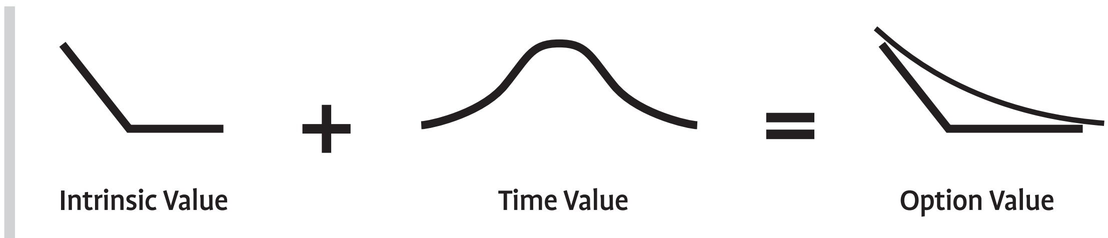


Long futures position benefits from price increases in the underlying, while losing value as the underlying decreases in price.



Short futures position benefits from price decreases in the underlying, while losing value as the underlying increases in price.

Option buyers pay a premium for the right of the option. This premium is made up of two parts, the intrinsic value and the time value. The intrinsic value of an option is the difference between the strike price and the market price. Upon expiry, the value of an option will be the maximum of 0 or the intrinsic value. The time value is a function of the cost of carry, the time to expiry, and the estimated market volatility.



Time value

Time value is assumed to be normally distributed around a mean (market price). It can be broken into three parts: the time to expiry, the cost of carry and the market volatility. Both the time to expiry and the fixed cost of carry are known (the basis of the CGBs). The third variable, set by the marketplace, is the magnitude of the time value (premium), which is the price volatility of the underlying.

Volatility

Price movements in the market can be measured against their movement away from a mean (average) price. A statistical analysis can be as rudimentary as a single linear regression analysis of past price action to far more complex geometrical translations of price trends in certain periods in time. Regardless of how models compare past price movement or historical volatility, pricing of options must encompass future volatility. This is why options may trade at a different volatility (market volatility) rate than the implied historical volatility.

Volatility smile

Option traders perceive the market as a normal distribution of possible outcomes. However, the out-of-the-money or deep-in-the-money options are priced with a “volatility smile”. This “smile” in a graphical form is the decreased volatility of at-the-money options being lower than the in-the-money or out-of-the-money options. Traders generally do not short these out-of-the-money options without demanding more premium.

Generally, there is always a 1% chance of a three standard deviation move in the price of the underlying.

Market volatility is the magnitude to which option participants will spend/take-in premium of an option given their hedging abilities. A high level of volatility presents more difficulty for hedging a short option position than when volatility is low.

Intrinsic value

The value of the payoff of an option at the time of expiry is the intrinsic value. The difference between the strike price of an option and the current market price will either be the positive (profit) or zero.

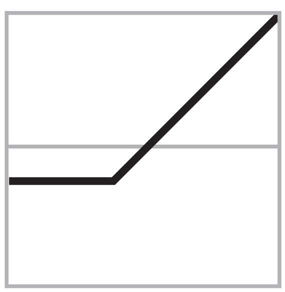
A 120 OGB call option which costs $1.40 has the following payoff. The premium paid for the option is a function of intrinsic value plus time value. When time value goes to zero (expiry), the call option payoff can be evaluated by discounting the call payoff by the price of the option. To experience a positive return on the money invested in the option, the market price must be higher than the strike by the cost of the option.

Strike	+	Price of option	=	Break-even level
120.00	**+**	**1.40**	**=**	**121.40**

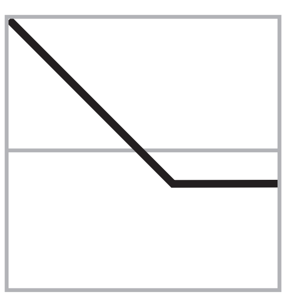
The break-even level on the 120 OGB calls is 121.40. This is the point where the return on the option position equals the investment in the option. The maximum loss is $1.40. The maximum gain is theoretically unlimited, as the price of the underlying can rise infinitely (though this outcome is not very likely).

MONEYNESS	INTRINSIC PRICE	CALLS	PUTS	ABSOLUTE DELTA[4]
In-the-money	Intrinsic positive	Strike < Market	Strike > Market	> 50%
At-the-money	Intrinsic near 0	Nearest strike to market	Nearest strike to market	50%
Out-of-the-money	Intrinsic 0	Strike > Market	Strike < Market	< 50%

Calls and puts



Call buyers have the right, but not the obligation, to buy a security at a fixed price in the future. They participate in the price appreciation of a security. If the price of the underlying increases, the price of the call will increase at an increasing rate. Similarly, a decrease in the price of the underlying will decrease the value of the option at a decreasing rate. The maximum downside of a call option buyer is the premium. The maximum upside is hypothetically unlimited.

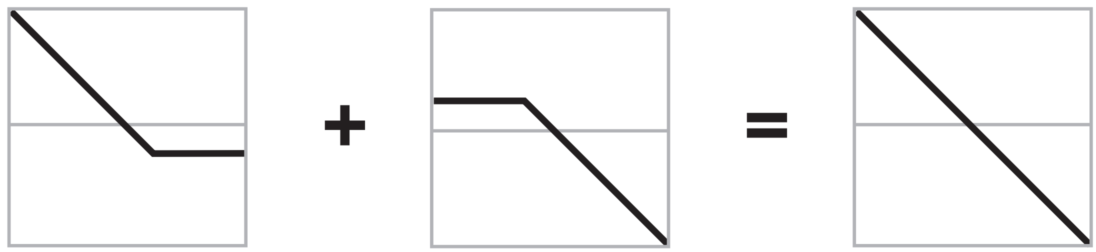


Put buyers have the right, but not the obligation, to sell a security at a fixed price in the future. They participate in the price depreciation of a security. If the price of the underlying decreases, the price of the put will increase at an increasing rate. Similarly, an increase in the price of the underlying will decrease the value of the option at a decreasing rate. The maximum downside of a put option buyer is the premium. The maximum upside is limited to the price of the security going to zero.

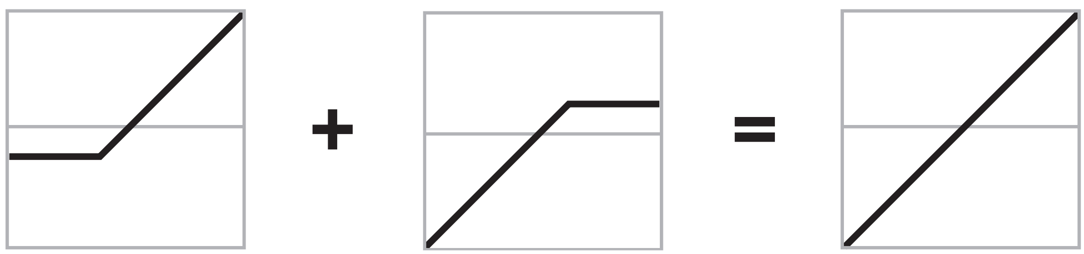
[4] See Section 9.

Put-call parity

Combining a long put position and a short call position is theoretically equal to a short futures position. Conversely, a long call position combined with a short put position is theoretically equal to a long futures position. When buying one option and selling another at the same strike, the time premium is both bought and sold. The remaining position is the expiry (intrinsic) payoff. The following diagrams detail this:

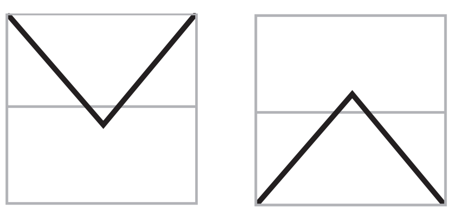


Long Put **Short Call** **Short Futures**



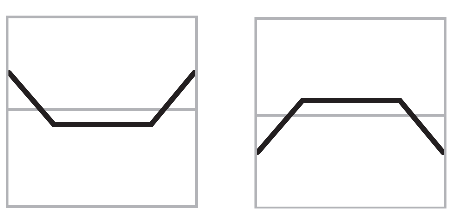
Long Call **Short Put** **Long Futures**

This parity allows a market participant great number of possible trading and hedging strategies.



Long Straddle **Short Straddle**

A straddle is a combination of a call and a put struck at the same price. A long straddle is the purchase of a call and a put; a short straddle is the sale of a call and a put.



Long Strangle **Short Strangle**

A strangle is a combination of an out-of-the-money call with an out-of-the-money put. A long strangle is the purchase of a call and a put; a short strangle is the sale of a call and a put.

The 5 Greeks

The pricing of an option as stated above is due to the cost of carry, the time to expiry and the estimated market volatility. From these variables, we can further examine the risk characteristics of an option contract. The following Greek letters are derived from the Black-Scholes option pricing model and represent the stochastic calculus derivatives with respect to the variables mentioned above.

Delta	is the change in the options price with respect to the change in the price of the underlying. Typically, this is stated as a percentage amount of the underlying asset. Both a long call and a short put position will have positive deltas. Conversely, a long put and a short call position will have negative deltas. The absolute value delta of an at-the-money option is 50%, out-of-the-money < 50% and in-the-money > 50%.
Gamma	is the change in the delta with respect to a change in the underlying asset. In stochastic calculus terminology, it is the second derivative of the price of an option with respect to the price of the underlying. Gamma typically grows larger for at-the-money options as the time to expiry draws closer.
Véga (Kappa)	is the change in an option price with respect to a change in the market volatility. Usually, this is stated in a one percent change in the volatility. The owner of an option contract is long volatility.
Theta	is the change in price of an option contract with respect to time. As the time to expiry draws closer, the time value of the option approaches zero. A long option position will lose its time value and thus option value. Institutional option traders further break theta into the two parts: daily accruals and the change in the value of the yield curve with respect to time.
Rho	is the change in the price of an option with respect to the risk-free rate of borrowing, or in the case of the CGBs the repo rate (represented by the basis of cash to futures). While this rate is integral to the pricing of the CGBs relative to the cash markets, the option price sensitivity is usually very low. Some multinational financial institutions break rho into financing risk (basis) and the cross-currency exposure of their reporting versus home currency.

Delivery

On the day of expiry, the option holder may choose to make or receive delivery, depending whether it is a call or put, for the CGB contract at the strike of the option. Option holders contact their broker to notify whether they will be making or receiving delivery. The brokers then inform the clearing house of the intentions of the option holder; the clearing house then informs the short option positions of their assignments. Typically, options contracts are automatically assigned if the options are in-the-money.

Margin

Margining options is similar to margining futures. A long option position need only pay the premium of the option, as the premium is the maximum downside of that position. Strategies combining several options require the amount of margin that will encompass the maximum downside of the strategy. If the maximum downside of an option strategy is more representative of an open futures position (i.e. short option position), the position will be margined as a combination futures and option position.

10. CDCC

Canadian Derivatives Clearing Corporation (CDCC), a wholly-owned subsidiary of the Montréal Exchange (MX), acts as the central clearing counterparty for exchange-traded derivative products in Canada and for a growing range of financial instruments trading in the over-the-counter (OTC) markets. CDCC's role is to ensure the integrity and stability of the markets that it supports.

CDCC occupies a unique space in the Canadian financial markets. Here are the main reasons behind CDCC's exclusive position:

- The only integrated central clearing counterparty in North America that clears and settles futures, options and options on futures.
- Thirty-five-year track record as the central clearing counterparty and guarantor of exchange-traded derivative products in Canada.
- AA rating from Standard & Poor's based on CDCC's prudent and standardized risk management policies and operational procedures.
- More than 30 clearing members, including major financial institutions and brokers in Canada.

11. Government of Canada Bond Futures Contract Specifications

	CGZ™ TWO-YEAR GOVERNMENT OF CANADA BOND FUTURES	CGF™ FIVE-YEAR GOVERNMENT OF CANADA BOND FUTURES
Contract Size	A Government of Canada bond having a nominal value of C$200,000.	A Government of Canada bond having a nominal value of C$100,000.
Delivery Standards	Government of Canada bonds which: have a remaining time to maturity of between 1 1/2 year and 2 1/2 years as of the first day of the delivery month, calculated by rounding down to the nearest whole month period; have an outstanding amount of at least C$2.4 billion nominal value; are originally issued at 2-year Government of Canada bond auctions; and, are issued and delivered on or before the 15th day preceding the first delivery notice day of the contract. Conversion factors computed with a yield equal to 6%.	Government of Canada bonds which: have a remaining time to maturity of between 3 1/2 years and 5 1/4 years as of the first day of the delivery month, calculated by rounding down to the nearest whole month period; have an outstanding amount of at least C$3.5 billion nominal value; are originally issued at 5-year Government of Canada bond auctions; and are issued and delivered on or before the 15th day preceding the first delivery notice day of the contract. Conversion factors computed with a yield equal to 6%.
Price Quotation	Par is on the basis of 100 points where one point equals to C$2,000.	Par is on the basis of 100 points where one points equals to C$1,000.
Price Fluctuation	0.005 = C$10	0.01 = C$10
Contract Months	March, June, September and December	
Last Trading Day	Seventh business day preceding the last business of the delivery month.	
Last Delivery Day	The last business day of the delivery month.	
Trading Hours	• Early session: 6:00 a.m. to 8:05 a.m. • Regular session: 8:20 a.m. to 3:00 p.m. • Extended session: 3:06 p.m. to 4:00 p.m. There is no extended session on the last trading day of the expiring contract month. During early closing days, the regular session closes at 1:00 p.m., time at which the daily settlement price is established. In those circumstances, the extended session is from 1:06 p.m. to 1:30 p.m.	
Ticker Symbols	**CGZ**	**CGF**

	CGB™ TEN-YEAR GOVERNMENT OF CANADA BOND FUTURES	LGB™ 30-YEAR GOVERNMENT OF CANADA BOND FUTURES
Contract Size	A Government of Canada bond having a nominal value of C$100,000.	A Government of Canada bond having a nominal value of C$100,000.
Delivery Standards	Government of Canada bonds which: have a remaining time to maturity of between 8 years and 10 1/2 years as of the first day of the delivery month, calculated by rounding down to the nearest whole three-month period; have an outstanding amount of at least C$3.5 billion nominal value; are originally issued at 10-year Government of Canada bond auctions; and are issued and delivered on or before the 15th day preceding the first delivery notice day of the contract. Conversion factors computed with a yield equal to 6%.	Government of Canada bonds which: have a remaining time to maturity of between 21 years and 33 years as of the first day of the delivery month, calculated by rounding down to the nearest whole three-month period; have an outstanding amount of at least C$3.5 billion nominal value; are originally issued at 30-year Government of Canada bond auctions; are issued and delivered on or before the 15th day preceding the first delivery notice day of the contract. Conversion factors computed with a yield equal to 6%.
Price Quotation	Per C$100 nominal value.	Per C$100 nominal value.
Price Fluctuation	0.01 = C$10	0.01 = C$10
Contract Months	March, June, September and December	
Last Trading Day	Seventh business day preceding the last business of the delivery month.	
Last Delivery Day	The last business day of the delivery month.	
Trading Hours	• Early session: 6:00 a.m. to 8:05 a.m. • Regular session: 8:20 a.m. to 3:00 p.m. • Extended session: 3:06 p.m. to 4:00 p.m. There is no extended session on the last trading day of the expiring contract month. During early closing days, the regular session closes at 1:00 p.m., time at which the daily settlement price is established. In those circumstances, the extended session is from 1:06 p.m. to 1:30 p.m.	
Ticker Symbols	**CGB**	**LGB**

OGB™

	OPTIONS ON TEN-YEAR GOVERNMENT OF CANADA BOND FUTURES
Contract Size	One Ten-Year Government of Canada Bond Futures (CGB)
Price Quotation	Quoted in points where each 0.005 point (0.5 basis point) represents C$5.
Price Fluctuation	0.005 = C$5
Contract Months	March, June September and December, plus two monthly contracts based on the next quarterly futures contract that is nearest to the options contract.
Strike Price Intervals	Set at a minimum of 0.5 point interval per CGB.
Last Trading Day	Third Friday of the month preceding the options contract month provided, however, that such Friday is a business day and precedes by at least two business days the first notice day of the underlying futures contract.
Exercise	Buyers of OGB options may exercise theirs options on any business day before cut-off time. The exercise cut-off time is 5:30 p.m. on the last trading day.
Expiration	At 11:59 p.m. on the last trading day.
Trading Hours	• Early session: 6:00 a.m. to 8:05 a.m. • Regular session: 8:20 a.m. to 3:00 p.m. • Extended session: 3:06 p.m. to 4:00 p.m. There is no extended session on the last trading day of the expiring contract month. During early closing days, the regular session closes at 1:00 p.m., time at which the daily settlement price is established. In those circumstances, the extended session is from 1:06 p.m. to 1:30 p.m.
Ticker Symbols	**OGB**

13. Strategies

In the following section, you will find bond future applications for each bond futures contract listed on the Montréal Exchange. These strategies have applications in corporate treasury and portfolio management, as well as the management of swaps books, mortgage loans, and asset-liability matching. These strategy examples are meant as a basic starting point that may be modified to suit the specific needs and constraints of a wide variety of different users.

C | G | Z ™

- Reducing the duration of a bond portfolio
- Lengthening the duration of a T-bill portfolio
- Yield curve spread Canada-U.S.
- Yield curve spread 2-year/10-year Government of Canada
- Hedging open swap positions
- Cash-and-carry trade
- Credit spread: 2-year Government of Canada bond futures / BAX 2-year strip

C | G | F ™

- Yield curve spread 5-year/30-year Government of Canada
- Cross hedging: Hedging a portfolio of Canada mortgage bonds (CMBs)

C | G | B ™

- Hedging a 10-year bond
- Synthetic bond portfolio
- Bond portfolio duration adjustment

L | G | B ™

- Reducing the duration of a bond portfolio
- Yield curve spread Canada-U.S.
- Yield curve spread 2-year/30-year Government of Canada
- Hedging open swap positions
- Cash-and-carry trade

strategies

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