While many Portfolio Managers and other fixed income traders analyze the relative value of bond futures using the net basis, fewer go the final step to compute the implied repo rate inherent in the differential between the futures price and the price of the deliverable bond.

Despite some trepidation on the part of futures market participants, calculation of the implied repo rate and the ensuing analysis need not be difficult and the benefits of calculating it can be quite valuable.

Benefits of calculating the implied repo for a contract include:

1. Net basis is usually a good proxy but the implied repo rate is the only completely accurate way to determine the cheapest-to-deliver bond.
2. The implied repo rate does not rely on the term repo rate or an assumption of the average repo rate on the bond between trade date and delivery date.
3. Modeling changes in the cheapest-to-deliver requires calculating the implied repo rate.
4. Net basis analysis alone cannot determine the optimal delivery date.
5. The implied repo rate can be compared to other short term investments.
6. Implied repo calculations can be useful in modeling a bond that will be deliverable into the contract but does not yet exist.
7. Implied repo rate is an excellent tool to analyze the "roll" between contracts near expiration.¹

**Simple Form of Implied Repo Rate**

Very simply, the repo rate implied in a futures contract is the yield one would earn by buying the cheapest to deliver bond at today’s price, simultaneously selling the futures contract, and delivering the bond to the contract buyer at some point during the delivery period.² The formula to calculate this metric is shown below in equation 1.

**EQUATION 1**

\[
R = \frac{(F)(CF) + AI_i - MV}{MV\ D/365}
\]

Where:

- R = Implied Repo Rate
- F = Futures contract price
- CF = Futures Conversion Factor
- AI_i = Accrued Interest on the bond at delivery
- MV = Market Value [dirty price] of the bond at purchase date
- D = Days between bond purchase and delivery

1. The "roll" will not be addressed in this paper as the CGF7 contract does not have enough open interest at this time to do a meaningful analysis.
2. Known as the Arbitrage Cash trade or method.
Adjusted for Interim Coupons

Since a coupon may sometimes be received on the cash bond between the purchase date and the delivery date, a minor complication needs to be introduced to account for the coupon receipt. An amended formula to account for coupons received between today and delivery is shown in Equation 2.

\[
EQUATION 2 \\
\begin{align*}
    r &= \frac{(F)(CF) + AI_2 - MV + C}{MV(D_1/365) - C(D_2/365)} \\
\end{align*}
\]

Where:

- \( R \) = Implied Repo Rate
- \( AI_2 \) = Accrued Interest on the bond at delivery
- \( F \) = Futures contract price
- \( MV \) = Market Value [dirty price] of the bond at purchase date
- \( CF \) = Futures Conversion Factor
- \( D_1 \) = Days between bond purchase and delivery
- \( C \) = Coupon received
- \( D_2 \) = Days between coupon receipt and futures delivery

Example Using CGF

To clarify the use of the above equation, we examine the December 2016 CGF contract using pricing on October 17, 2016.

- The delivery basket consists of the 0.75% Canada March 1, 2021; the 0.75% Canada September 1, 2021; and the 0.50% March 1, 2022 which have conversion factors of 0.8056, 0.7858, and 0.7554 respectively for the December CGF contract.
- First Notice day is November 28, 2016 and, using T+3 settlement for Canadian bonds greater than 3 years to maturity, First Delivery is December 1, 2016.
- Last Notice day is December 23, 2016 and, using T+3 and excluding 2 non-settlement holiday days, Last Delivery is December 30, 2016.
- Current prices of the securities in question are shown below:

<table>
<thead>
<tr>
<th>SECURITY</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75% Canada Mar. 1, 2021</td>
<td>100.156</td>
<td>100.177</td>
</tr>
<tr>
<td>0.75% Canada Sep. 1, 2021</td>
<td>99.989</td>
<td>100.028</td>
</tr>
<tr>
<td>0.50% Canada Mar. 1, 2022</td>
<td>98.381</td>
<td>98.414</td>
</tr>
<tr>
<td>CGFZ6 (CGF December 2016)</td>
<td>124.17</td>
<td>124.21</td>
</tr>
</tbody>
</table>

Using equation [2] we can calculate \( r \) for the Canada 0.75% March 2021 deliverable bond as shown below:

Where:

- \( R \) = Implied Repo Rate
- \( AI_2 \) = 0.2466 accrued interest on December 30, 2016
- \( F \) = 124.17 [bid side]
- \( MV \) = 100.177 clean [ask side] price + 0.1007 accrued interest on Oct. 20, 2016
- \( CF \) = 0.8056
- \( D_1 \) = 71 days between October 20, 2016 and December 30, 2016
- \( C \) = 0 [no coupon received]
- \( D_2 \) = Days between bond purchase and delivery

---

3. June and December CGB contracts require the more complex Implied Repo Rate calculation while March and September contracts do not. Similarly, March and September CGF contracts require a coupon receipt adjustment while June and December contracts do not.


6. Source: CanDeal

7. Source: Montreal Exchange
r = \frac{[124.17](0.8056) + 0.2466 - (100.177 + 0.1007) + 0}{(100.177+0.1007) (71/365) - 0 (0/365)}

r = \frac{(100.0314) + 0.2466 - (100.277)}{(100.277) (71/365)}

r = \frac{0.0003}{19.5061} = 0.0013\%

Following the above process for each of the deliverable bonds on the first delivery date and the last delivery date, we compute the results in Table 1.

<table>
<thead>
<tr>
<th>BOND</th>
<th>Implied Repo to Dec 1, 2016</th>
<th>Implied Repo to Dec 30, 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN 0.75% Mar. 2021</td>
<td>-0.51%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CAN 0.75% Sep. 2021</td>
<td>-20.56%</td>
<td>-11.86%</td>
</tr>
<tr>
<td>CAN 0.50% Mar. 2022</td>
<td>-40.23%</td>
<td>-23.59%</td>
</tr>
</tbody>
</table>

Comparison to Other Short Term Investments

Since the implied repo on the futures contract, given the October 17th pricing, indicates a yield of 0% to December 30, 2016, it is unlikely that an investor with cash to invest would choose to do so by buying the cheapest-to-deliver bond and waiting to deliver it on December 30th. A cursory examination of similar fixed income products with greater liquidity\(^8\), such as a Canada T-Bill that matures December 29 has a yield of 0.50%, far better than the 0% that the cash arbitrage on the CGF December contract would generate.

An implied repo rate lower than that offered on similar investments is normal in futures contracts since the seller of the contract retains certain options for delivery\(^9\) that have value. However, at current low yields, the value of those options is significantly diminished.

Of course, the calculation above is the best outcome for a non-leveraged investor who attempts the Arbitrage Cash method on this contract. Both the other bonds that are not the cheapest-to-deliver but which are deliverable into the December CGF contract result in a significant loss regardless of whether the investor chooses to deliver at the beginning of December or the end.

Optimal Delivery

After calculating the implied repo rate once, further modeling of the optimal delivery time by calculating the implied repo rate for each possible delivery date in the delivery period becomes trivial as shown in Figure 1 below. If an investor was actually long the bond and short the contract, he or she would prefer to make the highest return possible which indicates that optimal delivery is on the final day of the delivery period, or December 30, 2016.

---

8. Liquidity for the Arbitrage Cash method should be evaluated by examining both the bid/ask on the round-trip trade as well as the open interest on the futures contract.
9. Both the quality option (i.e. which bond to deliver) as well as the timing option.
Evolution of the Cheapest-to-Deliver Bond

A further analysis can be performed where the investor is interested in whether or not a change in the cheapest-to-deliver bond is likely and under what scenarios it will change. This analysis can be conducted by assuming an instantaneous parallel change in yields and calculating the implied repo rate for each bond in the deliverable basket at the new yield. A comparison of implied repo rates can then be made to determine whether a different bond is cheaper to deliver given the changed yield levels.

Table 2 depicts this analysis for 25 basis point incremental changes in yield all the way up to +200 bps. Note that the implied repo rates (or gains to the Arbitrage Cash trade) are not the gains that will be realized by the Portfolio Manager if rates change. The Arbitrage Cash trader will lock in the return associated with the implied repo rate at inception, not a theoretical implied repo rate assuming a selloff in bond yields. One can easily observe at this point that the CAN 0.75% Mar 2021 bond would remain the cheapest-to-deliver even in a severe selloff.

<table>
<thead>
<tr>
<th>yid Δ (bps)</th>
<th>-50</th>
<th>-25</th>
<th>0</th>
<th>25</th>
<th>50</th>
<th>75</th>
<th>100</th>
<th>125</th>
<th>150</th>
<th>175</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAN 0.75% Mar. 2021</td>
<td>-10.91%</td>
<td>-5.48%</td>
<td>0.00%</td>
<td>5.53%</td>
<td>11.12%</td>
<td>16.75%</td>
<td>22.44%</td>
<td>28.18%</td>
<td>33.98%</td>
<td>39.83%</td>
<td>45.73%</td>
</tr>
<tr>
<td>CAN 0.75% Sep. 2021</td>
<td>-23.70%</td>
<td>-17.81%</td>
<td>-11.86%</td>
<td>-5.84%</td>
<td>0.24%</td>
<td>6.38%</td>
<td>12.58%</td>
<td>18.86%</td>
<td>25.19%</td>
<td>31.60%</td>
<td>38.07%</td>
</tr>
<tr>
<td>CAN 0.50% Mar. 2022</td>
<td>-36.38%</td>
<td>-30.02%</td>
<td>-23.59%</td>
<td>-23.59%</td>
<td>-10.50%</td>
<td>3.83%</td>
<td>2.91%</td>
<td>9.73%</td>
<td>16.64%</td>
<td>23.62%</td>
<td>30.69%</td>
</tr>
</tbody>
</table>

Summary

Implied repo analysis can add an extra dimension of accuracy and depth to analysis of futures positions and the deliverable basket. This paper has explored three of these analyses and shown Portfolio Managers and other futures traders how to use the implied repo rate of a futures contract to compare an Arbitrage Cash trade to other short term investments, to calculate the optimal delivery date, and to analyze the evolution of the cheapest-to-deliver bond.

10. However, in the event of a switch in the cheapest-to-deliver bond (almost impossible in this example), delivering the cheaper bond will augment returns for the Arbitrage Cash trade.
11. In fact, the shortest to maturity bond remains the cheapest-to-deliver until rates approach 6%.